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LETTER TO THE EDITOR

On the critical dynamics of the diluted Q-state Potts models

S Jain

Department of Theoretical Chemistry, University Chemical Laboratory, University of Cambridge, Lensfield Road, Cambridge CB2 1EW, UK

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Abstract. The results of Monte Carlo simulations of the four-state Potts model on a square lattice at the bond percolation threshold are presented. Estimates are given for the new dynamical exponents A and B. Our result for A is in clear contradiction with a recent conjecture of Nunes da Silva and Lage.

A recent [1] neutron scattering experiment on $Rb_2Co_pMg_{1-p}F_4$, a site-diluted two-dimensional Ising antiferromagnet, near the percolation threshold, $p_c = 0.5927$, has generated considerable interest [2-8] in the critical dynamics of diluted spin systems.

In the dynamic scaling hypothesis the average relaxation time, $\tau_{AV}(T)$, scales as

$$\ln(\tau_{AV}(T)) = f(\ln \xi_T) \tag{1}$$

where ξ_T is the thermal correlation length. Conventional dynamic scaling [9] would imply that $f(\ln \xi_T) \sim Z(\ln \xi_T)$, where Z is a dynamic critical exponent. For percolating systems ($p = p_c$), however, it is found [2-8] that Z is no longer a constant but a temperature-dependent function

$$Z(T) = A(\ln \xi_T) + B. \tag{2}$$

Very recently, the breakdown of dynamic scaling has also been confirmed for two-dimensional diluted Potts models [6, 7] and the bond-diluted Ising model on hypercubic lattices [8]. The analytic work of Nunes da Silva and Lage [7, 8] is particularly interesting because it suggests that the new dynamical exponents depend very strongly on both the dimensionality, d, and the number of spin components, q.

The dimensionality and symmetry of the problem are two crucial factors governing the critical behaviour of a system. By varying just the number of spin components and keeping fixed all other features of the system (the type of dilution, the updating rate, etc) we are able to investigate the q dependence of the dynamic behaviour.

In this letter we present the results from Monte Carlo simulations of the two-dimensional four-state Potts model on a square lattice at the bond percolation threshold. We shall show that our results contradict Nunes da Silva and Lage [7].

We choose the Hamiltonian to be

$$H = -\sum_{\langle ij \rangle} J_{ij} \delta_{\alpha_i \alpha_j} \tag{3}$$

where $\alpha_i (\alpha_i = 1, \ldots, q)$ are the Potts spins situated on every site of a 64×64 square lattice ($q = 4$ in our simulations) and the nearest-neighbour ferromagnetic couplings are selected according to

$$P(J_{ij}) = \tfrac{1}{2}[\delta(J_{ij}) + \delta(J_{ij} - 1)]. \tag{4}$$

Imposing periodic boundary conditions, we update the spins via the Metropolis transition probability. The data were collected over the temperature range $0.44 \leq T/T_c(\text{pure}) \leq 1.10$ and have been averaged over 10–248 samples; the largest statistical error bar is less than 10%. The only difference between the system discussed here and the one simulated by Jain *et al* [6] is the number of Potts states (this should be consulted for further technical details).

We define an average relaxation time by

$$\tau_{AV} = \tfrac{4}{3} \int_0^{+\infty} \left(N^{-1} \sum_i \delta_{\alpha_i(t_0)\alpha_i(t+t_0)} - \tfrac{1}{4} \right) \mathrm{d}t \tag{5}$$

where $N = 4096$, the number of spins, and $t = t_0$ indicates an equilibrium state of the system. The asymptotic behaviour of the spatial correlation function,

$$\Gamma(n) = \tfrac{4}{3}\left(N^{-1} \sum_i \langle \delta_{\alpha_i \alpha_{i+n}} \rangle_T - \tfrac{1}{4} \right) \tag{6}$$

where $\langle \ldots \rangle_T$ implies a thermal average and n ($n = 0, 1, \ldots, 10$) is the displacement in the x direction, is given by

$$\Gamma(n) \sim \exp(-n/\xi_T) \qquad \text{for} \qquad n \gg \xi_T. \tag{7}$$

Equation (7) enables us to extract the thermal correlation length for any temperature. Now, one expects [10]

$$\xi_T(q) = \xi_0(q) \exp(\beta\nu) \tag{8}$$

where $\beta = 1/T$, $\nu(=\nu_T)$ is the (universal) thermal exponent and $\xi_0(q)$ is the non-universal amplitude.

In figure 1 we show a plot of $\xi_T(q=3)/\xi_T(q=4)$ against T. The weighted line of best fit indicates that $\xi_0(q=3)/\xi_0(q=4) = 1.20 \pm 0.11$. Since [6] $\xi_0(q=3) = 0.16 \pm 0.02$, we have that $\xi_0(q=4) = 0.14 \pm 0.03$. Further, if we assume that $\nu = \frac{4}{3}$, the conjectured theoretical value [9], then we have that for all temperatures simulated $\xi_T \ll$ linear size

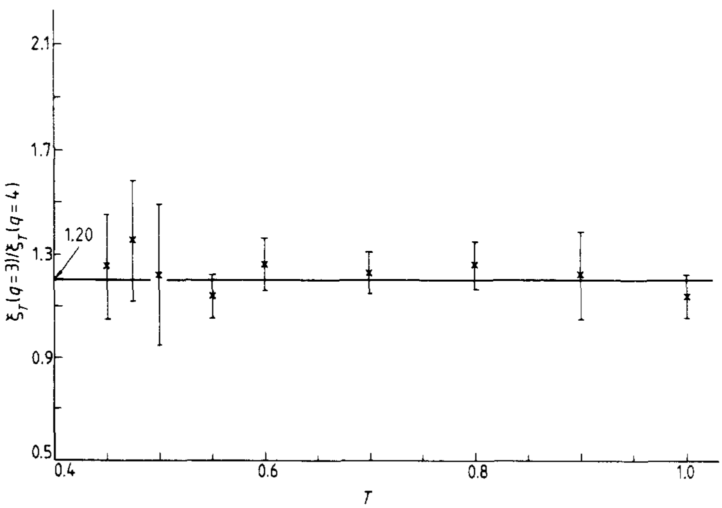

Figure 1. A plot of $\ln \xi_T(q=3)/\ln \xi_T(q=4)$ against T. The intercept on the y axis implies that $\xi_0(q=3)/\xi_0(q=4) = 1.20 \pm 0.11$.

of the lattice and $\xi_T \gg 1$ for $T \leqslant 0.625$. So our results are not expected to be influenced by finite-size effects [4].

Writing

$$\ln \tau_{AV} = Y_1/T^2 + Y_2/T + Y_3 \tag{9}$$

we have that $A(q) = Y_1/\nu^2$, $B(q) = \nu^{-1}(Y_2 - 2Y_1\nu^{-1} \ln \xi_0(q))$ and $Y_3 =$ constant. In figure 2 we show $\ln \tau_{AV}$ against $1/T$. The best quadratic fit to the data for $T \leqslant 0.625$ gives $A(q=4) = 0.56 \pm 0.04$ and $B(q=4) = 4.76 \pm 0.80$. Table 1 contains the various estimates which have been made for $A(q)$ and $B(q)$ in two dimensions. We note that the value of $A(q=4)$ is not consistent with the conjecture of [7] who suggest that $A(q=4)/A(q=2) = \frac{3}{2}$.

We can make a more direct comparison with the $q=3$ case by considering

$$\ln[\tau_{AV}(q=3)/\tau_{AV}(q=4)] = \alpha_1/T^2 + \alpha_2/T + \alpha_3 \tag{10}$$

where $\alpha_1 = \nu^2[A(q=3) - A(q=4)]$, $\alpha_2 = \nu\{2[A(q=3)\xi_0(q=3) - A(q=4)\xi_0(q=4)] + (B(q=3) - B(q=4))\}$ and $\alpha_3 =$ constant. Thus, $A(q=3) = A(q=4)$ would imply that a plot of $\ln[\tau_{AV}(q=3)/\tau_{AV}(q=4)]$ plotted against $1/T$ should be linear. Such a plot is shown in figure 3. The line of best fit for $T \leqslant 0.625$ has slope 1.00 ± 0.22 and intercept -1.75 ± 0.30. The data presented in table 1 are consistent with a slope in the range

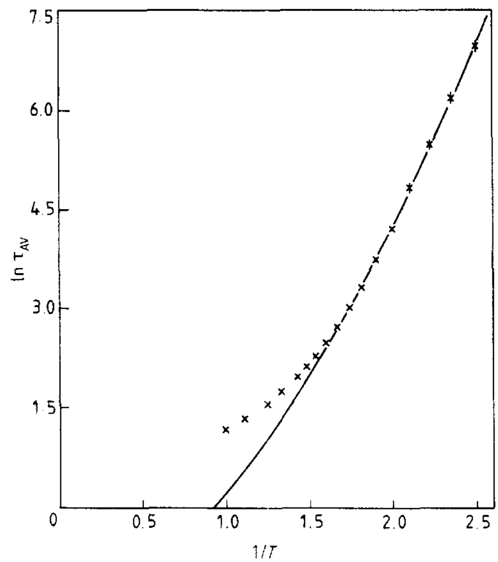

Figure 2. A plot of $\ln \tau_{AV}$ against $1/T$ for $q=4$. The quadratic fit shown yields $A(q=4) = 0.56 \pm 0.04$ and $B(q=4) = 4.76 \pm 0.80$.

Table 1. The behaviour of $A(q)$ and $B(q)$ for $d=2$.

Reference	$A(q)$	$B(q)$
Jain [4]; $q=2$	0.51 ± 0.05	3.25 ± 0.41
Jain *et al* [6]; $q=3$	0.78 ± 0.15	3.35 ± 0.88
This work; $q=4$	0.56 ± 0.04	4.76 ± 0.80
Nunes da Silva and Lage [7]	$A(q)/A(2) = 2(q-1)/q$	

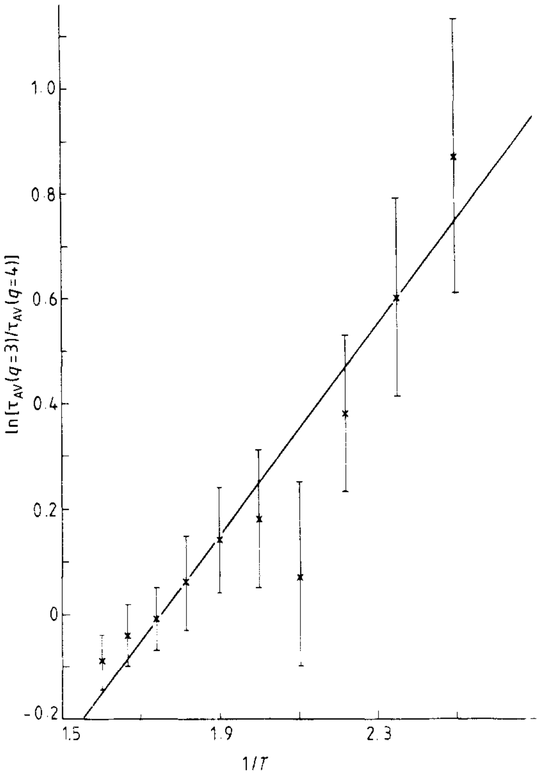

Figure 3. A plot of $\ln[\tau_{AV}(q=3)/\tau_{AV}(q=4)]$ against $1/T$. The straight line, which has slope 1.00 ± 0.22, is a consequence of assuming that $A(q=3) = A(q=4)$.

−4.19 to 0.53. Note that, if we also have $B(q=3) = B(q=4)$, then the slope would be expected to be 0.05 ± 0.12. So it would appear that A is possibly independent of q. B, however, probably depends on the number of Potts states.

To conclude, we have given estimates for the new dynamical exponents A and B for the four-state Potts model at the bond percolation threshold. Our results, although in agreement with Jain *et al* [6], contradict a recent conjecture of Nunes da Silva and Lage [7].

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References

[1] Aeppli G, Guggenheim H and Uemura Y J 1984 *Phys. Rev. Lett.* **52** 942
[2] Henley C L 1985 *Phys. Rev. Lett.* **54** 2030
Harris C K and Stinchcombe R B 1986 *Phys. Rev. Lett.* **56** 869
Stinchcombe R B 1985 *Scaling Phenomena in Disordered Systems* ed R Pynn and A Skjeltorp (New York: Plenum)
Lage E J S 1986 *J. Phys. C: Solid State Phys.* **19** L91
[3] Rammal R and Benoit A 1985 *J. Physique Lett.* **46** L667; 1985 *Phys. Rev. Lett.* **55** 649
[4] Jain S 1986 *J. Phys. A: Math. Gen.* **19** L57, L667
[5] Chowdhury D and Stauffer D 1986 *J. Phys. A: Math. Gen.* **19** L19

Pytte E 1986 *Phys. Rev.* B **34** 2060
[6] Jain S, Lage E J S and Stinchcombe R B 1986 *J. Phys. C: Solid State Phys.* **19** L805
[7] Nunes da Silva J M and Lage E J S 1987 *J. Phys. A: Math. Gen.* **20** 2655
[8] Nunes da Silva J M and Lage E J S 1987 *J. Phys. C: Solid State Phys.* **20** L275
[9] Hohenberg P C and Halperin B I 1977 *Rev. Mod. Phys.* **49** 435
[10] Stinchcombe R B 1983 *Phase Transitions and Critical Phenomena* vol 7, ed C Domb and J L Lebowitz (New York: Academic) p 151